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The Nonlinear Mathematical Modeling and Optimization of Distributed Control in Complex Systems

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Abstract: With the widespread application of complex systems in industries such as manufacturing, transportation, and energy, their high-dimensional, strongly nonlinear, and dynamically coupled characteristics pose significant challenges to traditional centralized control. To address these complexities more efficiently, this study constructs a nonlinear mathematical model by introducing nonlinear feature mapping into a multiple linear regression framework and implements distributed optimization using the Alternating Direction Method of Multipliers (ADMM). The proposed method is validated through the simulation of the nonlinear dynamic behavior of a deep-water riser–test pipe system, with experimental designs encompassing multi-dimensional vibration responses and dynamic environmental disturbances. The results demonstrate that the proposed nonlinear model significantly outperforms other methods in terms of prediction accuracy and optimization efficiency. Under varying amplitudes and frequencies of disturbances, the model achieves lower error rates and higher robustness, with an adaptation decay rate of less than 17.6%. These findings indicate that the proposed nonlinear modeling and distributed optimization approach can effectively capture the dynamic characteristics of complex systems, making it suitable for real-time distributed control scenarios with promising engineering applications.

Keywords: Complex Systems; Multiple Linear Regression Model; Nonlinear Features; Distributed Optimization; Optimization Efficiency.

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1. Introduction

With the continuous development of science and technology, the application of complex systems in industries such as manufacturing, transportation, energy, and information technology has been expanding rapidly [1]. These systems are often characterized by high dimensionality, strong nonlinearity, dynamic coupling, and multiple constraints, making traditional centralized control methods increasingly inadequate to meet practical needs. Due to its fl exibility, scalability, and computational parallelization, distributed control has emerged as a key research focus for the control of complex systems [2]. However, research on distributed control in complex systems still faces many challenges. The dynamic relationships and nonlinear coupling characteristics within complex systems are highly intricate, and traditional mechanism-based analytical methods often struggle to construct mathematical models that align with real-world scenarios. This frequently results in decreased control performance and computational efficiency. Additionally, some optimization methods exhibit high computational complexity when addressing distributed nonlinear systems, making them unsuitable for real-time control requirements. Consequently, constructing efficient and accurate nonlinear mathematical models combined with advanced optimization algorithms has become a core research direction in addressing the distributed control problems of complex systems [3].

In this context, leveraging large-scale data processing technologies widely applied in complex systems allows the modeling of system dynamics through data-driven approaches. Distributed optimization algorithms can further enhance the performance and efficiency of control systems. By designing well-constructed nonlinear mathematical models, it becomes possible not only to capture the nonlinear dynamic characteristics within systems but also to provide theoretical support and engineering guidance for real-time fault prediction and parameter optimization in complex systems. Such methods can effectively identify potential risk factors, enable early parameter adjustments, reduce the risk of control system failures, and provide reliable technical means to ensure the safety and efficient operation of systems.

2. Nonlinear mathematical modeling and optimization

2.1 Nonlinear mathematical modeling strategy

In complex systems, due to the highly nonlinear coupling relationships between internal variables, directly establishing precise mathematical models is often challenging. However, by applying appropriate linear transformations to nonlinear systems, certain nonlinear problems can be converted into analytically solvable linear problems, thereby simplifying the modeling and solution process [4-5]. This approach is widely used in nonlinear regression modeling, where constructing suitable feature spaces or performing variable transformations enables the simplification of originally complex nonlinear relationships into linear forms.

Taking into account the linear transformation within nonlinear systems, a standard representation of a nonlinear system model is given as:

$$\begin{aligned} y(k) = f\big(&y(k-1),...,y(k-n), \\ &u(k-1-k_d),...,u(k-n_u-k_d)\big)+d(k) \end{aligned} \tag{1}$$

Where: $f(\bullet)$ represents the unknown linear function, $u(\bullet)\in R^m$ is the control input vector, $y(\bullet)\in R^q$ is the system output vector, n_u and n_y denote the input and output orders. k_d is the delay and $d(k)$ is the noise. The nonlinear system's mathematical model operates in a rolling manner. At each sampling time k, a nonlinear optimization problem needs to be solved online to obtain the control effect. Assuming the constraint optimization performance index $J(k)$ at time k is defined as::

$$\begin{aligned} J(k) = &\frac{1}{2}\sum_{h=1}^{N_p}\left[r(k+h|k)-y(k+h|k)\right]^2 + \\ &\frac{1}{2}\sum_{h=1}^{N_u} q_h \Delta u^2(k+h-1|k) \end{aligned} \tag{2}$$

The constraints are:

$$u_{\min} \le u(k+h-1|k) \le u_{\max} \tag{3}$$

$$\Delta u_{\min} \le \Delta u(k+h-1|k) \le u_{\max} \tag{4}$$

$$y_{\min} \le y(k+h|k) \le y_{\max} \tag{5}$$

Where: $r(k+h|k)\in R^q$ is the expected output of the h th step starting from sampling time k , and $y(k+h|k)\in R^q$ is the predicted output of the h th step starting from sampling time k.

And the control increment is:

$$\Delta u(k+h-1|k) = u(k+h-1|k) - u(k+h-2|k) \tag{6}$$

Where: N_p and N_u represent the prediction and control time domains, respectively. q_h is the control weight coefficient, $u_{\max}$ and $u_{\min}$ define the bounds of $u(k+h-1|k)$. $\Delta u_{\max}$ and $\Delta u_{\min}$ specify the bounds of $\Delta u(k+h-1|k)$. $y_{\max}$ and $y_{\min}$ are the upper and lower bounds of $y(k+h|k)$.

Distributed control in complex systems needs to address highly coupled nonlinear dynamic characteristics, and nonlinear mathematical modeling serves as the core foundation for achieving efficient control. Multiple linear regression models demonstrate strong capabilities in handling large-scale data computation and prediction, and they can capture nonlinear relationships within systems by introducing nonlinear features or variable transformations. Therefore, based on multiple linear regression models, we can integrate nonlinear feature expansions or other nonlinear transformation methods to construct nonlinear mathematical models suitable for complex systems. This enables better characterization of the system's dynamic behavior and optimization of control performance.

2.2 Model Optimization

In the distributed control of complex systems, model optimization is a critical step in enhancing system performance and achieving efficient control. Generally, optimization strategies for nonlinear mathematical models can be broadly categorized into two types: model optimization methods based on physical mechanisms and data-driven model optimization methods.

2.2.1 Mechanism-based model optimization

The mechanism-based analysis method relies on an in-depth understanding of system characteristics, analyzing causal relationships and extracting the internal mechanism of the system to construct a mathematical model with clear physical significance. Models optimized using this method typically have high interpretability and applicability, making them more suitable for achieving collaborative optimization in distributed control systems.

During the model optimization process, it is essential to first clarify the system context and modeling objectives. This involves analyzing the actual operational background of the complex system and defining the primary goals of model optimization, such as improving prediction accuracy, reducing computational complexity, or enhancing real-time performance. Based on the system's characteristics, reasonable simplifications and assumptions should be made, discarding secondary factors and emphasizing the role of key variables. This process requires combining the system's underlying mechanisms with data analysis to extract critical features, ultimately linearizing or nonlinearizing the problem to provide support for subsequent optimization.

The focus of model optimization lies in the rational construction of nonlinear relationships, analyzing the causal relationships among system variables, and designing mathematical structures that align with real-world scenarios by integrating the system's mechanisms. In distributed control, the model must undergo error analysis and robustness verification to ensure its stability and accuracy during actual operation.

2.2.2 Data-driven model optimization

For complex systems characterized by strong nonlinearity, time variability, and uncertainty, data-driven model optimization methods have increasingly become a significant research focus. Unlike traditional mechanism-based approaches, data-driven methods directly construct or optimize nonlinear mathematical models by collecting and processing large amounts of system operation data, thereby enhancing system prediction capabilities and control effectiveness. By analyzing data, these methods identify key variables that critically influence system control while eliminating irrelevant factors to reduce modeling complexity. Optimization algorithms are employed to dynamically adjust the parameters of the nonlinear model, improving its adaptability and accuracy [6]. During model operation, continuous error analysis is conducted, and dynamic optimization is performed based on distributed feedback mechanisms. By improving the model structure or refining the optimization algorithm, the model can more accurately predict the system's dynamic behavior. This reduces model-solving time and meets the real-time requirements of distributed control systems. Additionally, through error analysis and model adjustments, the model maintains stability in complex and dynamic environments.

2.2.3 Optimize the process

The nonlinear mathematical modeling and optimization of distributed control in complex systems require a combination of mechanism-based analysis and data-driven approaches, with continuous optimization built on the foundation of model construction. Through reasonable simplifications, error correction, and the application of distributed optimization algorithms, the prediction accuracy and control performance of the model can be effectively enhanced, providing strong support for distributed control in complex systems. Figure 1 illustrates the optimization process, which proceeds as follows:

(1)Data Collection and Preprocessing

Collect large amounts of sample data from the complex system, including input vectors, state variables, and output results. Clean and preprocess the data by removing outliers, filling in missing values, and normalizing the data to improve quality.

(2)Initial Model Construction

Use data-driven methods or mechanism-based approaches to construct an initial nonlinear mathematical model. Introduce specific nonlinear feature mappings into the model to describe the system's nonlinear dynamic characteristics.

(3)Parameter Optimization

Apply fitting algorithms to perform an initial optimization of the model parameters to obtain output results. Continuously adjust input vectors, control output vectors, and calculate the model's predictions.

(4)Model Validation

Compare the prediction results with expected values, calculate errors, and analyze their sources. Validate the model's rationality and applicability and assess its performance in real-world systems.

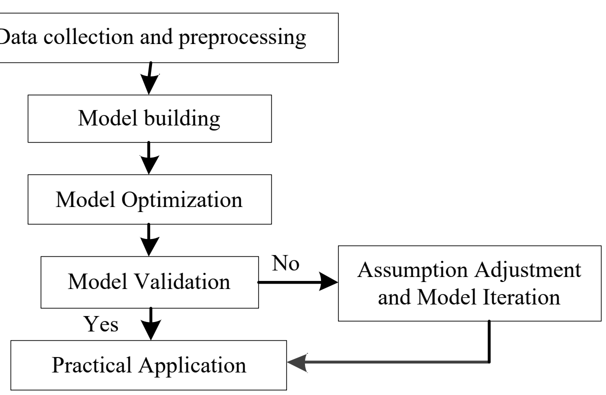
(5)Assumption Adjustment and Iterative Improvement

Based on validation results, check for deficiencies in the model assumptions. If the model does not fully meet practical requirements, revise or supplement the assumptions and rebuild the model. Through multiple rounds of optimization and adjustments, progressively refine the model until the prediction results achieve the desired accuracy.

(6)Application in Distributed Control

Apply the final optimized nonlinear mathematical model to distributed control systems for real-time prediction, fault detection, or resource allocation. Dynamically update the model parameters during operation to adapt to system changes.

Figure 1 Optimization process



3 Experimental results and analysis

In complex systems, nonlinear mathematical modeling and optimization of distributed control usually require experimental verification of the effectiveness of modeling and the applicability of the optimization process. In order to verify the advantages of the constructed nonlinear mathematical model in terms of prediction accuracy and optimization performance, this paper designs a distributed vibration control experiment based on the dynamic behavior of complex systems.

3.1 Experimental Configuration

The experiment selected dynamic vibration control in complex systems as the research object, simulating the nonlinear dynamic behavior of the deepwater riser-test pipe system to verify the prediction accuracy and optimization effect of the nonlinear mathematical model. The system structure is shown in Figure 2. The riser subsystem includes vibration behaviors in the cross-flow direction and the downstream direction, which is used to simulate the dynamic impact of environmental disturbances on complex systems. The test pipe subsystem includes vibration responses in the downstream and upstream directions, which is used to simulate the dynamic coupling relationship between multiple subsystems.

3.2 Experiment variable settings

The vibration frequency is the output variable of the system, which is used to measure the dynamic response of the riser-test tube system under external disturbance. In this experiment, the vibration frequency range is set to 0.5Hz to 10Hz to cover the common low-frequency vibration characteristics in deepwater environments. The data sampling frequency is 1000Hz to ensure the capture of the detailed characteristics of the system vibration process. The independent variable setting includes the response quantities in four vibration directions, and the specific ranges are as follows:

(1) Transverse vibration is the reflection of the lateral force of the fluid on the riser, which is mainly caused by waves and lateral disturbances of the fluid. In the experiment, the displacement range of transverse vibration is set to ±10mm, the initial vibration frequency is 2Hz, and the disturbance intensity gradually increases.

(2) Streamwise vibration reflects the movement characteristics of the riser in the direction of water flow, and the displacement range is set to ±15mm, and the corresponding disturbance frequency is 1Hz to 5Hz.

(3) Streamwise vibration is mainly affected by the coupling effect of the riser vibration, and the displacement range is ±12mm. During the test, the flow rate is gradually increased (0.5m/s to 1.5m/s) to analyze the coupling effect under different flow conditions.

(4) The countercurrent vibration is mainly caused by fluid backflow disturbance, with a displacement range of ±8mm, and the corresponding flow rate range is set to 0.3m/s to 1.2m/s.

To ensure the environmental consistency and rationality of the structural characteristics of the experiment, Table 1 shows the specific parameters of the auxiliary variables. Fluid density and flow rate are the key control variables of the experiment, which directly affect the vibration response characteristics of the riser and test pipe. The length, diameter and stiffness coefficient of the riser and test pipe comprehensively reflect the physical characteristics of the experimental system. The experiment lasted for 60s, combined with a sampling frequency of 1000Hz, which can ensure that dynamic data is collected for a long enough time. The high-frequency sampling rate can also capture the subtle vibration changes of the system under complex dynamic conditions.

Table 1 Specific parameters of auxiliary variables

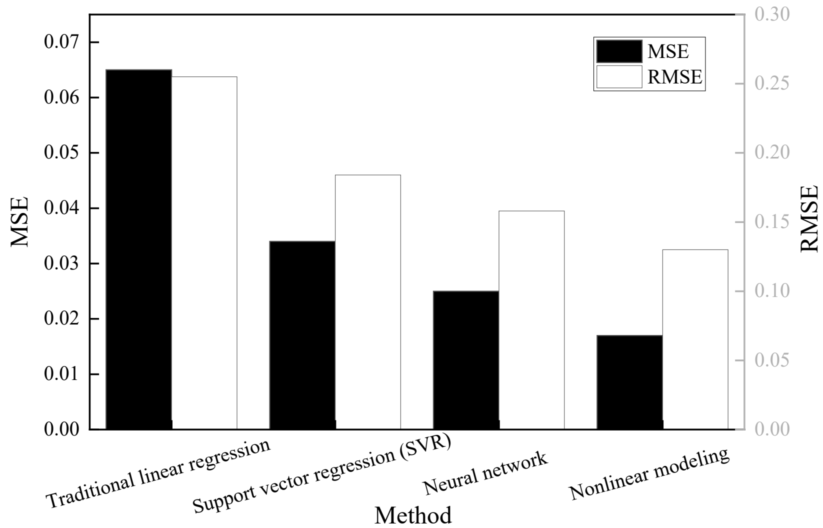
Category	Variable name	Numeric
Environmental parameters	Fluid density	1025 kg/m³
	Flow rate	0.3 m/s-1.5 m/s
	Fluid temperature	10°C -20°C
Structural parameters	Riser length	100 m
	Riser diameter	0.5 m
	Riser stiffness factor	2000 N/m
	Test tube length	80 m
	Test tube diameter	0.3 m
	Test tube stiffness factor	1500 N/m
Time Dimension	Duration of each experiment	60s
	Data sampling frequency	1000 Hz
	Number of data sampling points	60000

3.3 Results Analysis

3.3.1 Prediction Error

The data collected in the experiment contains 60,000 records, which are derived from the time series data of each experiment. The sampling frequency is 1000 Hz and the experiment duration is 60 s. Figure 3 shows the model prediction accuracy evaluation, using mean square error (MSE) and root mean square error (RMSE) as the main evaluation indicators. Compared with the traditional linear regression model, the proposed nonlinear mathematical model has an MSE and RMSE of 0.017 and 0.130, respectively, which significantly improves the prediction accuracy. Compared with SVR and neural network, the MSE and RMSE of the proposed method are reduced, indicating that the model is superior in capturing nonlinear characteristics. The proposed nonlinear modeling method can efficiently capture the nonlinear dynamic characteristics of the system in complex systems, while having both prediction accuracy and computational efficiency, and is very suitable for the design and optimization of real-time distributed control systems.

Figure 3 Model prediction accuracy evaluation



3.3.2 Optimizing efficiency

In the nonlinear modeling of distributed control in complex systems, the convergence speed and computation time of optimization algorithms are key metrics for evaluating their efficiency. This experiment compares the proposed nonlinear mathematical model with traditional linear regression, support vector regression (SVR), and neural networks. Table 2 presents the results of the optimization efficiency comparison. The proposed nonlinear mathematical model achieves optimization with the fastest convergence speed, requiring only 30 iterations to meet the convergence condition. In contrast, traditional nonlinear regression models require 150 iterations, and the step size for each update must be carefully adjusted; otherwise, issues such as oscillations or slow convergence may arise. SVR and neural networks require 50 and 100 iterations, respectively, and while both perform well in handling nonlinear problems, their heuristic optimization methods lead to slower convergence processes due to the randomness of the initial population.

The proposed nonlinear mathematical model has a total computation time of 10.2 seconds, significantly outperforming other methods and demonstrating the notable advantages of distributed optimization under parallel computation. Although traditional nonlinear regression models involve simpler computations, their excessive iteration count results in a total time of 28.5 seconds. SVR and neural networks require 18.7 seconds and 33.4 seconds, respectively, as their computation involves extensive population updates and fitness evaluations, leading to higher computational complexity. The significant advantages of the proposed nonlinear mathematical model in terms of convergence speed and computation time, as validated through comparisons with other optimization methods, highlight its superiority in nonlinear modeling and optimization for complex systems.

Table 2 Comparison results of optimization efficiency

Method	Convergence iterations	Convergence time/s	Calculate total time/s
Traditional linear regression	150	6.8	28.5
Support vector regression (SVR)	50	4.2	18.7
Neural network	100	7.6	33.4
Nonlinear modeling	30	2.5	10.2

3.3.3Robustness

To evaluate the model's adaptability under different input disturbances, the experiment introduced noise with varying amplitudes and frequencies into the test data to simulate real-world input disturbances. The disturbance amplitudes were set at ±5%, ±10%, and ±15% of the input signal values, representing mild, moderate, and severe disturbances. The results of the anti-disturbance capability test are shown in Table 3. For the nonlinear modeling method, the MSE under low-amplitude disturbances was only 0.020, close to 0.017 in the undisturbed scenario, with an adaptation decay rate (AR) of 17.6%, demonstrating strong robustness. In contrast, the MSE for traditional linear regression increased rapidly under disturbance conditions, reaching 0.130 at ±15% amplitude and 10 Hz frequency, indicating poor adaptability to nonlinearity and disturbances. Under all disturbance conditions, the AR of the nonlinear modeling method was significantly lower than that of traditional linear regression and SVR, showing its superior ability to handle input disturbances. The nonlinear modeling method effectively reduces the impact of input disturbances on prediction results, maintaining high prediction accuracy, especially under low-amplitude disturbances. This indicates strong robustness and adaptability for distributed control in complex systems. Traditional linear regression, due to its inability to capture nonlinear relationships, shows poor adaptability to disturbances and is unsuitable for complex dynamic environments. Although neural network methods demonstrate better adaptability, their low computational efficiency makes them less suitable for real-time control scenarios.

Table 3 Anti-interference ability test results

Method	Disturbance Amplitude	MSE (perturbation)	MSE (undisturbed)	Fitness decay rate (AR)
Traditional linear regression	±5%	0.06	0.045	33.30%
	±10%	0.095	0.045	111.10%
	±15%	0.13	0.045	188.90%
Support vector regression (SVR)	±5%	0.03	0.025	20.00%
	±10%	0.04	0.025	60.00%
	±15%	0.055	0.025	120.00%
Neural network	±5%	0.018	0.015	20.00%
	±10%	0.024	0.015	60.00%
	±15%	0.032	0.015	113.30%
Nonlinear modeling	±5%	0.02	0.017	17.60%
	±10%	0.026	0.017	52.90%
	±15%	0.034	0.017	100.00%

4. Conclusion

This study investigates the nonlinear mathematical modeling and optimization of distributed control in complex systems and proposes an efficient modeling method that combines nonlinear feature mapping with distributed optimization. Based on the experimental results, the following conclusions can be drawn:

(1) The proposed nonlinear mathematical model, integrating feature mapping with multiple linear regression, effectively captures the nonlinear dynamic characteristics of complex systems. The MSE and RMSE reached 0.017 and 0.130, respectively, which are significantly better than those of traditional linear regression, SVR, and neural network methods.

(2) The nonlinear mathematical model exhibits significant advantages in optimization speed and computational efficiency, requiring only 30 iterations to converge with a computation time of 10.2 seconds, far lower than SVR (18.7 seconds) and neural networks (33.4 seconds).

(3) Under input signal disturbance conditions, the model demonstrates strong robustness. At a disturbance amplitude of ±5%, the adaptation decay rate is 17.6%, which is significantly lower than that of other comparison methods.

The proposed nonlinear modeling and optimization approach combines prediction accuracy with computational efficiency, making it suitable for real-time control, fault prediction, and resource allocation scenarios in complex systems. It provides critical theoretical support for the engineering practices of distributed control.

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Conflict of Interests

The author(s)declare(s) that there is no conflict of interest regarding the publication of this paper.

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