Asia Pacific Science Press

Asia Pacific Economic and Management Review
ISSN(O):3005-9275
ISSN(P):3005-9267
Vol. 2 No.5 (2025)

A Concise Algorithm for Calculating the Number of Microstates in a Bose System

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Abstract: By introducing a model of “small balls separated by boards”, this paper transforms the method for solving the number of microstates of a Bose system and presents an algorithm that is more concise than the one provided in textbooks. Studies have shown that “Thermodynamics·Statistical Physics” is a relatively difficult specialized course; learners often encounter concepts that are hard to understand and complex calculations during their study. However, the calculation method for the number of microstates of a Bose system presented in Wang Zhicheng’s book is not concise enough and tends to raise questions among students. Through analysis, this paper points out the shortcomings of the calculation method in Wang Zhicheng’s book. By elaborating on the concise algorithm proposed by the authors, it clearly demonstrates that this algorithm makes the calculation process of the number of microstates of a Bose system more intuitive and can avoid some doubts existing in textbooks. Finally, the physical connotation of the number of microstates of the Bose system can also be clearly observed through our algorithm.

Keywords: Thermodynamic and Statistical Physics; Number of Microstates; Boltzmann System

Published: Sept 22, 2025

DOI: https://doi.org/10.62177/apemr.v2i5.597

1.Introduction

In the study of Statistical Physics, distribution and microstates are difficult points for students. It is very important to understand and master the derivation method of the number of microstates. However, in the actual teaching process, some students think that the calculation process of the number of microstates of the Bose system in Wang Zhicheng’s book “Thermodynamics·Statistical Physics” is difficult to understand [1], and there are several doubtful points that need to be clarified. Therefore, it is necessary to propose a more concise method for calculating the number of microstates of the Bose system. As early as 1997, someone proposed a method for calculating the number of microstates of a classical system different from that in textbooks [2]. In 2003, Song Weicai used the metaphor of “people checking into a hotel” to understand the calculation of the number of microstates of the Bose system in textbooks and provided an explanation for the unreasonable parts of the algorithm in textbooks [3]. In 2019, Lan Shanquan proposed two different algorithms for the distributions of the three systems, but they were still not concise enough [4]. In 2025, Li Yuanchang et al. solved the problem of the thermodynamic probability of N gas molecules distributed in the left and right halves of a rectangular container and quantitatively verified Boltzmann’s statistical explanation of the microscopic nature of the second law of thermodynamics [5]. These works reflect the importance of solving the number of microstates of the system [6][7]. On this basis, we propose a concise and intuitive method for calculating the number of microstates of the Bose system based on previous studies [8].

2.The classical method for calculating the number of microstates of a Bose system

First, it is necessary to calculate the number of possible ways for a_l particles to occupy ω_l quantum states at the energy level ϵ_l. Use ① , ② , ... to represent each quantum state, and ○ to represent each particle. Arrange them in a row, with the leftmost position fixed as quantum state ① . Since there are a total of $\omega_l + a_l - 1$ quantum states and particles except quantum state ① , there are $(\omega_l + a_l - 1)!$ ways to arrange them. Then, divide by the number of mutual exchanges between all particles $a_l!$ and the number of mutual exchanges between quantum states $(\omega_l - 1)!$, so there are $(\omega_l + a_l - 1)!/[a_l!(\omega_l - 1)!]$ possible ways. Multiply the results of each energy level, and the number of microstates corresponding to the distribution $\{a_l\}$ of the Bose system is obtained [1]

$$\Omega_{B.E} = \Pi_l \frac{(\omega_l + a_l - 1)!}{a_l!(\omega_l - 1)!}$$

Although the above conclusion appears in textbooks, there are several doubtful points that need to be clarified: (1). Why should the number of arrangements be divided by the number of mutual exchanges between quantum states $(\omega_l - 1)!$. If the arrangement remains the same after the exchange of quantum states, does it mean that each quantum state at the same energy level is indistinguishable? Is this contradictory to the assumption that the degeneracy is ω_l? 2. The number of arrangements of ω_l quantum states should also be $\omega_l!$ instead of $(\omega_l - 1)!$. The explanation in the book is that the leftmost position is fixed as quantum state ① , but what is the reason? No explanation is given in the textbook. The above problems easily cause doubts among students, so is there a better method to solve this problem?

3.A Concise Algorithm

Consider a_l identical particles occupying ω_l quantum states at the energy level ϵ_l. For ease of understanding, we can model this as placing a_l indistinguishable small balls into ω_l boxes and arranging them in a row, where each such arrangement represents a way of particles occupying each quantum state. For example, the arrangement in Figure 1 shows that 1 particle is occupied on quantum state 1, 3 particles on quantum state 2, and 2 particles on quantum state 3. Since there is no limit to the number of particles occupying a single quantum state in a Bose system, there is no limit to the relative values of a_l and ω_l. In this model, there is also no limit to the number of small balls in each box. To eliminate the influence of exchanging boxes (quantum states) on the arrangement of this problem, we equate the model to using $\omega_l - 1$ boards to separate a_l small balls. This is because $\omega_l - 1$ boards are needed to divide the small balls into ω_l groups (Figure 2). Similarly, the arrangement in Figure 2 shows that 1 particle is occupied on quantum state 1, 3 particles on quantum state 2, and 2 particles on quantum state 3. Therefore, Figure 1 and Figure 2 represent equivalent problems. Now let's look at Figure 2: even if the boards are exchanged, it will not affect the distribution of small balls in each corresponding quantum state. In other words, although the ω_l quantum states at the energy level ϵ_l are not identical, the boards separating these quantum states in this model can be regarded as identical. This is because whether the partitions are the same or not does not affect the grouping of small balls; the partitions only serve to make the grouping clearer.

Figure1: particle occupying on quantum state

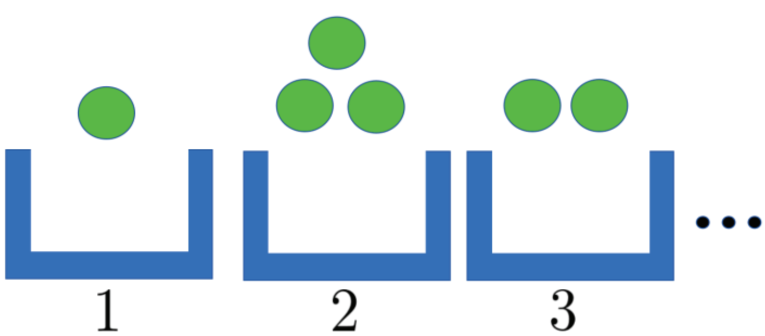
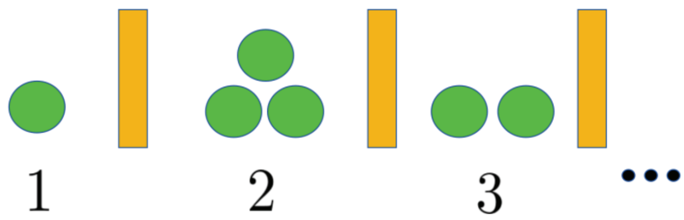


Figure2: our equivalengt model



Next, this model can be used to directly calculate the number of microstates of the Bose distribution. Number the a_l small balls as 1,2,3,...,a_l, and number the $\omega_l - 1$ boards as 1,2,3,...,$\omega_l - 1$. Then arrange them in a row. Exchanging the relative positions of the partitions and the small balls is equivalent to rearranging the system composed of the small balls and the boards. Thus, the total number of small balls plus boards (the number of particles plus the number of quantum states) is $a_l + \omega_l - 1$, and there are $(\omega_l + a_l - 1)!$ ways to arrange them. Since the small balls are completely identical (particles in a Bose system are indistinguishable), we should also divide by the number of mutual exchanges between the small balls (particles) $a_l!$ and the number of mutual exchanges between the boards $(\omega_l - 1)!$ (obviously, exchanging the boards will not change the distribution of particles in different quantum states). Therefore, there are $(\omega_l + a_l - 1)!/[a_l!(\omega_l - 1)!]$ possible ways for a_l particles to occupy ω_l quantum states at the energy level ϵ_l. Similarly, since this is the arrangement method for one energy level, the results of each energy level are finally multiplied. Therefore, the number of microstates corresponding to the distribution $\{a_l\}$ of the Bose system is calculated as follows

$$\Omega_{B.E} = \Pi_l \frac{(\omega_l + a_l - 1)!}{a_l!(\omega_l - 1)!}$$

This calculation result is consistent with the result in Wang Zhicheng's book, but it is more concise and will not cause misunderstandings among readers.

4.Discussion

By introducing the model of "inserting boards among small balls", we propose a completely new method for calculating the number of microstates of a Bose system. This method can avoid the problems existing in the method from Wang Zhicheng's textbook, such as:

The textbook does not clearly state whether the quantum states at the same energy level are indistinguishable. This premise is of great importance. If the quantum states are distinguishable, the calculation result does not need to be divided by the number of mutual exchanges between quantum states, which contradicts the calculation result in the textbook; if the quantum states are indistinguishable, this is inconsistent with the basic assumption of the Bose system (that quantum states are distinguishable). However, in our algorithm, the boards that separate these quantum states can be regarded as identical, and exchanging the boards will not affect the distribution of small balls in each corresponding quantum state, thus avoiding this problem.

2. The textbook does not provide a reason for fixing the leftmost end of the arrangement as quantum state ① . Based on this, the textbook gives the arrangement of the remaining quantum states as $(\omega_l - 1)!$, and this process is very forced and unconvincing. Our algorithm exactly requires using $\omega_l - 1$ boards to separate a_l small balls (because dividing a_l small balls into ω_l groups requires $\omega_l - 1$ boards), and the arrangement of $\omega_l - 1$ boards is exactly $(\omega_l - 1)!$with no contradictions.

Conclusion

By introducing the "boards inserting into small balls" model, we propose a new calculation method that can well avoid these problems. This method is not only simple in the calculation process but also helps teachers and students better understand the physical connotation of the Bose distribution.In conclusion, by introducing the model of inserting boards among small balls to replace the original arrangement, we propose a completely new and concise algorithm for calculating the number of microstates of a Bose system. Compared with the algorithm in the textbook, our algorithm avoids the confusion caused by directly arranging particles and quantum states. Furthermore, it has the advantages of a simple calculation process and easier understanding. We have reason to believe that mastering this algorithm can help teachers and students better understand the physical connotation of the Bose distribution.

Funding

No

Conflict of Interests

The authors declare that there is no conflict of interest regarding the publication of this paper.

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